Methods for Calculating the Subsonic Aerodynamic Center of Finite Wings

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Introduction

THE aerodynamic center (a.c.) is a fundamental parameter in aeroelastic analyses and is critical in predicting flutter and divergence speeds. Lifting surface theories can be evaluated by their ability to predict aerodynamic centers in agreement with experimental data. A standard method in use for subsonic speeds is the vortex-lattice method (VLM). Its modern version divides the wingspan into strips and each strip is divided into lifting elements (boxes). The earliest version was proposed by Campbell in 1951 (Ref. 1) and it used only a single horseshoe vortex on each strip with its bound leg at the strip quarter-chord and it matched the downwash at the three-quarter-chord on the centerline of each strip. Later developments, such as those by Hedman[2] and Belotserkovskii,[3] considered multiple boxes on each strip placing the bound legs on the box quarter-chords and matching the downwashes at the box centerline three-quarter-chords These three-dimensional approximations are based on the success in two dimensions of choosing the one-quarter- and three-quarter-chord points to match the exact two-dimensional airfoil theory. The three-dimensional VLM can be expected to predict an a.c. near the quarter-chord of a finite wing because the basic lifting element (the box) has its a.c. at the quarter-chord (because the bound vortex is placed there) and agrees with experimental data.[4]

Constant Pressure Panel Method

In place of the aforementioned conventional vortex system, recent papers by Liu and his associates[5,6] have proposed the lifting element to be a constant-pressure panel with its collocation point chosen empirically near its trailing edge (at 85% of the box chord). It has been suggested[7] that this constant-pressure panel method (CPPM) is more accurate because it is a higher order method and should be more robust in terms of modeling the surfaces into boxes. However, one would not expect the CPPM to lead to a quarter-chord aerodynamic center because the fundamental lifting element has its a.c. at its 50% chord. It is the purpose of this Note to show that the higher order CPPM is less accurate than the VLM and that the calculated a.c. can lie well behind the quarter-chord when the number of chordwise boxes is small.

We choose a rectangular wing to illustrate both methods. We consider an aspect ratio (AR) of 20 with the wing pitching about its 50% chord and flying at a Mach number (M) of zero. We idealize the wing into $N_S = 10$ and 40 strips and consider several chordwise boxes, $N_C = 1, 3, 5, 10,$ and 15, on each strip. The calculations for the VLM are based on the N5KQ version of the doublet-lattice method[8] in MSC/NASTRAN.[9] (Note that N5KQ reduces to the VLM at zero reduced frequency and $k = 0.001$ is used.) The calculations for the CPPM are based on the ZONA6 option in ZAERO.[10] The results for the various values of N_C and N_S are shown in Table 1.

A perusal of the table shows a consistent prediction of the quarter-chord a.c. by the VLM and that the CPPM is consistently aft. Of course, with only one box on the strip the VLM predicts the quarter-chord exactly, whereas the CPPM predicts the 50% chord exactly!

Table 1 Static longitudinal characteristics of rectangular wing with AR = 20 at $M = 0$

Divisions		C_{x_α}		$(C_{m_\alpha})_{0.5}$		a.c. (% chord)	
N_C	N_S	N5KQ	ZONA6	N5KQ	ZONA6	N5KQ	ZONA6
1	10	−5.386	−6.337	1.346	0.0	25.00	50.00
	40	−5.341	−6.234	1.335	0.0	25.00	50.00
3	10	−5.392	−5.637	1.359	1.055	24.79	31.28
	40	−5.351	−5.546	1.351	1.041	24.76	31.23
5	10	−5.392	−5.585	1.360	1.200	24.77	28.52
	40	−5.429	−5.495	1.372	1.184	24.73	28.46
10	10	−5.392	−5.561	1.360	1.302	24.77	26.59
	40	−5.430	−5.471	1.372	1.284	24.72	26.53
15	10	−5.392	−5.557	1.361	1.335	24.77	25.97
	40	−5.430	−5.467	1.373	1.317	24.72	25.92

As the number of boxes is increased the VLM always predicts a slightly forward a.c. but the CPPM approaches the 25% chord very slowly (not in a robust manner!).

Conclusions

Because of the aft a.c., the CPPM will lead to *unconservative* predictions of flutter or divergence speeds. The error in a.c. is not measured from the leading edge but from the elastic axis (EA). A popular example to demonstrate aeroelastic analyses is the Bisplinghoff, Ashley, and Halfman (BAH) wing,[11] which has an EA at its 35% chord. [A direct aerodynamic comparison between MSC/NASTRAN[9] and ZAERO[10] utilizing the BAH wing (MSC/NASTRAN Example HA145B) could not be performed because the ZAERO beam spline (SPLINE2) cannot represent a "stick" model.] Considering the results shown in Table 1 for the model with 5 chordwise boxes and 10 strips we compare the VLM a.c. at 24.77% to the CPPM a.c. at 28.52%. This is a difference of 38% when measured from the EA at 35%. If we consider the results from the large model with 10 chordwise boxes and 40 strips we have the VLM a.c. at 24.72% and the CPPM a.c. at 26.53%. This is a smaller difference of 18% when again measured from the EA at 35%. However, even the smaller error is unacceptable when one considers that the margin of safety prescribed by the Federal Aviation Regulations[12] to prevent flutter and divergence is only 15%!

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